MA3025 Exam # 2

Due 9am/11am November 15th, 2007 **Name** ______________________________

Instructor: Dr. Ralucca Gera

Show all necessary work in each problem to receive credit. Please turn in well-organized work and complete solutions. You may ONLY use your notes and Rosen book (no collaboration is allowed either). The maximum score for the exam is 100.

1. (10 points) True or false (no need to justify):

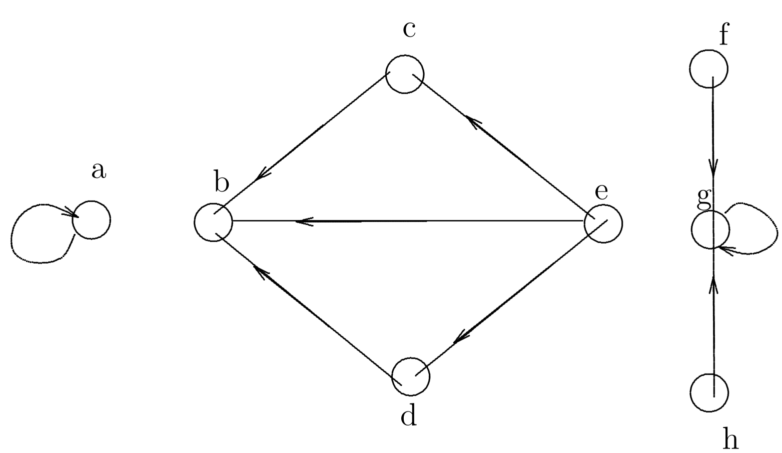
(a) Every 2×2 matrix with a nonzero determinant has an inverse.

(b) The recurrence $a_n = 2a_{n-1} + \sqrt{2}a_{n-2} + \pi a_{n-3}$ with $a_0 = 0$, $a_1 = 2$ and $a_2 = 3$ is a linear homogeneous recurrence with constant coefficients of degree 3.

(c) The equation $a_n = (n-1)!, n \geq 1$ is a solution to the recurrence $a_n = n \cdot a_{n-1}, n \geq 2$ with $a_1 = 1$.

(d) Is the relation given by the following matrix symmetric? $\begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$.

(e) Is the relation given by the following digraph transitive?



2. (15 points) Recall that the Fibonacci sequence F_n is defined by $F_0 = 0, F_1 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$. The Lucas sequence L_n is similarly defined, by $L_0 = 2, L_1 = 1$, and $L_n = L_{n-1} + L_{n-2}$ for $n \geq 2$. (The two sequences use the same recurrence, but with different initial conditions.) Prove that, for all $n \geq 2$ we have that $5F_{n+2} = L_{n+4} - L_n$.

3. (15 points) Let L_n be defined by $L_0 = 2, L_1 = 1$, and $L_n = L_{n-1} + L_{n-2}$ for $n \geq 2$. Prove that for nonnegative integers n we have that $\sum_{i=0}^{n} L_i = L_{n+2} - 1$.

4. (10 points) Solve $a_n = \frac{a_{n-2}}{9}$ for $n \geq 2$ with $a_0 = 0$ and $a_1 = 1$

5. (10 points) Find a recurrence for the relation $a_n = (-1)^n n!$ for $n \geq 0$. Simplify as much as possible.

6. (10 points)

(a) Find the number of terms in the formula for the number of elements in the union of 4 sets given by the principle of inclusion-exclusion. Some terms may be zero, and you should count them as well.

(b) How many bit strings of length 14 do not contain 12 consecutive 1s?

2 extra credit points Find the number of terms in the formula for the number of elements in the union of 40 sets given by the principle of inclusion-exclusion. Some terms may be zero, and you should count them as well.

7. (10 points) Let $A = \{-1, 0, 1, 2, 3, 4\}$ and $R = \{(-1, 2), (-1, 3), (0, 0), (1, 1), (2, 3)\}$.

(a) Find R^2 and R^3

(b) is the element $(1, 3)$ in R^{2007}?

(c) list each of R, R^2 and R^3 with a matrix

(d) draw the directed graphs that represent R, R^2 and R^3.

8. (20 points) let A be the set of all binary strings of length 100. Define a relation R on A by $(x, y) \in R$ if the binary strings x and y agree in the first and the last bit. Answer with explanations if R:

(a) reflexive?

(b) irreflexive?

(c) symmetric?

(d) antisymmetric?

(e) transitive?

(f) Find the equivalence classes if they exist.

(g) How many elements are there in each of the equivalence classes above?

(h) Do the classes form a partition? If so, what are they a partition of? If not, what set should they partition?

(i) How many elements are there in the relation R above?

(EXTRA CREDIT: 5 points) Let R be a relation defined on the set of integers by the division property:

$$R = \{(a, b) : a|b\}$$

Is R:

(a) reflexive?

(b) symmetric?

(c) antisymmetric?

(d) transitive?

(e) Find the equivalence classes if they exist.